"Are there any tips to help me to understand how a surface will look?"

That's a very good question!

Most Surfer users start from a known formula, say f(x, y, z), which gives a simple surface, and then change it by turning and cutting it with the vertical slide. If you insert the letters a, b, c or d in the formula, then you can choose them and thus easily change the surface with the horizontal slides.

Examples:

1. z=0 is the equation of the (x, y) plane, i.e. the surface of the points with the coordinates x, y, z for which is z=0. In this case f(x, y, z) is simply z.
2. x^2+y^2=1 is the equation of the circle with the radius 1 in the (x, y) plane. However, given a three-dimensional space with the coordinates x-y-z, z can be anything. For each z, as a result, you get a circle of the radius 1 and altogether it equals a cylinder (a pipe) of radius 1. The formula of the pipe thus reads x^2+y^2-1.
3. If you want to change the diameter, in Surfer you select e.g. the parameter a. Then the formula x^2+y^2-a^2 provides the cylinder of the radius a. (If, for example, you turn the image so that the z-axis points vertically backwards you see only the circle of the cylinder.)
4. x^2+y^2+z^-a^2 is the formula of the spherical surface of a ball of radius a. The formula x^2+b*y^2+z^2-a^2 describes an ellipsoid (an egg shape) if b is not 1. and x^2-b*y^2+z^2-a^2 describes a hyperboloid (an hour-glass shape). Note that Surfer shows all images within an invisible sphere whose size is controlled by the vertical scrollbar. If you change this bar this egg e.g. turns into a napkin ring (if the bounding sphere cuts the egg) and suddenly nothing remains to be seen (if the bounding sphere disappears in the egg).

In addition there are three important rules:

A. Several surfaces:
If you multiply two formulas f and g you get, by f*g=0, the unification of the two surfaces f=0 and g=0. The new surface is singular along the intersection f=g=0. For example x*(x^2+y^2+z^2-1) =0 is the union of the plane with the sphere and that looks (within the bounding sphere) like Saturn with a ring. Surfer enables you to generate and merge the surfaces individually. This reduces the complexity of calculation and allows more varied colourings. However, the merging of components (see B) requires the product.

B. Merging of components:
If you subtract a constant from a formula f, say a, then the surface f(x, y, z) =0 becomes f(x, y, z) -a=0, in particular, the singularities of f(x, y, z) =0 are smoothed.

For example, if the union of two surfaces given as a product f*g is changed to f*g-a, then the surface is smoothed along the join of the separate surfaces, and the two components f=0 and g=0 merge into one component.

This smoothing process can be well observed if you gradually increase the value a in the formula x*(x^2+y^2+z^2-1)-a starting from 0.

C. Section curves:

If f=0 and g=0 are the formulas of two surfaces f^2+g^2=0 is the formula of the intersection as this equation is equivalent to f=g=0. The intersection will not be seen, though, because it is one-dimensional and the visualization software cannot see it.

The trick is to "thicken" the intersection by f^2+g^2-a, a small value for a will make it visible.

These three principles enable you to create many beautiful surfaces. The more basic shapes (in addition to plane, sphere, ellipsoid etc.) you can think of, the more complicated surfaces you can create.

Tips given by Prof. Gert-Martin Greuel, director of the Mathematisches Forschungsinstitut Oberwolfach and the Imaginary exhibition.